7: Errors, Statistical Analysis, and Spreadsheets

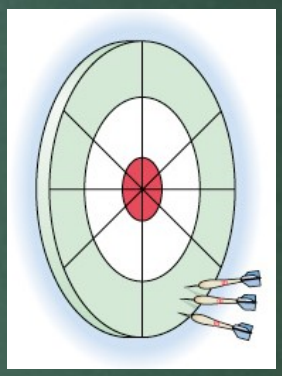
- Errors
- Uncertainty and Error Propagation
- Statistical Analysis
- Spreadsheets

Announcements / Reminders

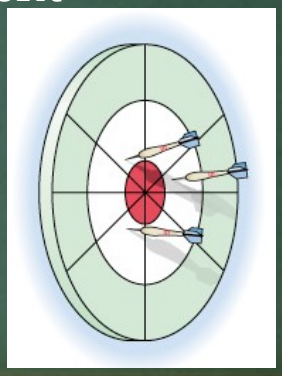
- Mid-Term (Exam 1) will be a Take-Home
  - Available via Moodle and website on Friday, Mar 14
  - Due 5PM Tuesday, Mar 25
  - Must work independently!
- Homework for Lab 6 will be due with Lab 7
  - Ch 3 and Ch 10

Types of Errors

- Systematic Error—will get the same "wrong" answer
  - Ex: un-leveled balance

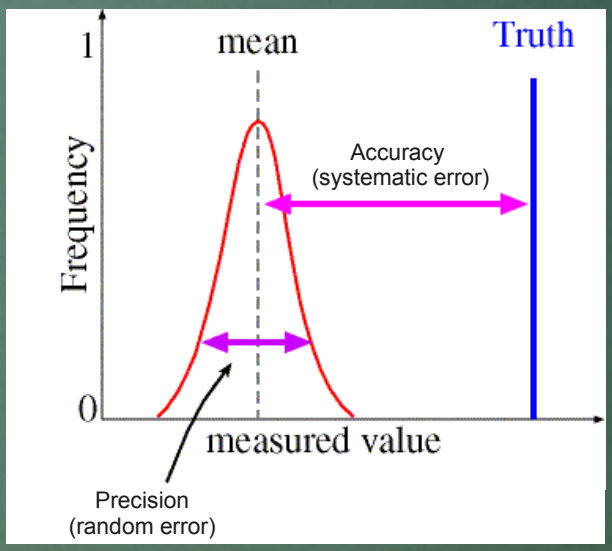


- Random Error—will get slightly different results every time
  - "noise" or "uncertainty"



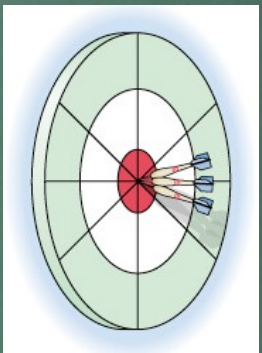
Precision and Accuracy

- Accuracy: Ability to get the "true" answer
- Precision: Reproducibility of the result



"Fixing" Errors

- Systematic: Calibrate instruments, compare to "knowns"
- Random: Repeat experiment, use Statistical Analysis



Ideally want both: accuracy and precision

No Errors!

Uncertainty

EVERY measurement has uncertainty associated with it

Example: Volumetric Flasks:

Volume	Tolerance (Class A)
1 mL	0.01 mL
10 mL	0.02 mL
25 mL	0.03 mL
50 mL	0.05 mL
100 mL	0.08 mL
250 mL	0.10 mL
500 mL	0.15 mL
1000 mL	0.30 mL

Absolute Uncertainty

Volumetric Flasks:

Volume	Tolerance (Class A)
1 mL	0.01 mL
10 mL	0.02 mL
25 mL	0.03 mL
50 mL	0.05 mL
100 mL	0.08 mL
250 mL	0.10 mL
500 mL	0.15 mL
1000 mL	0.30 mL

1.00 ± 0.01 mL

100.00 ± 0.08 mL

Percent Relative Uncertainty

Volumetric Flasks:

$$\frac{\text{Absolute uncertainty}}{\text{Magnitude of Measurement}} \times 100\%$$

Volume	Tolerance (Class A)
1 mL	0.01 mL
10 mL	0.02 mL
25 mL	0.03 mL
50 mL	0.05 mL
100 mL	0.08 mL
250 mL	0.10 mL
500 mL	0.15 mL
1000 mL	0.30 mL

1.00 ± 1% mL

100.00 ± 0.08% mL

Propagating Uncertainty

If two or more uncertain numbers are added, subtracted, multiplied, or divided, what is uncertainty of answer?

Adding/Subtracting:

$$e_4 = \sqrt{e_1^2 + e_2^2 + e_3^2}$$

Multiplying/Dividing:

$$\%e_4 = \sqrt{(\%e_1)^2 + (\%e_2)^2 + (\%e_3)^2}$$

Examples of Propagating Uncertainty

Volume delivered from buret = Final vol – Initial vol

$\pm\sqrt{(0.02)^2+(0.02)^2}$ mL
$= 0.028$ mL

± 0.02 mL ± 0.02 mL

Calculating Concentration = $\frac{\text{mol}}{\text{Vol}}$ ± 0.03 % ± 0.1 %

$\pm\sqrt{(0.03\%)^2+(0.1\%)^2}$
$= 0.1\%$

Statistical Analysis

- Analyze data from repeated experiments
- Find “outliers” and invalid data
- Determine if data from 2 different sources are “different”

Statistical Analysis: “Simple” Functions

	Name	Formula	Excel Function
1	Number of Observations	n	COUNT(data)
2	Mean	$\bar{x}=\frac{1}{n}(x_1+x_2\cdots x_n)$	AVERAGE(data)
3	Standard Deviation	$s=\sqrt{\frac{\sum_i (x_i-\bar{x})^2}{n-1}}$	STDEV(data)
4	Minimum value		MIN(data)
5	Maximum value		MAX(data)

Statistical Analysis

6	Student's *t*	See Table 4-2 (pg 87)	TINV(α, df)
7	Confidence interval	$\mu_{CL} = \bar{x} \pm \frac{ts}{\sqrt{n}}$	AVERAGE(data) + CONFIDENCE(α, *s*, *n*) AVERAGE(data) - CONFIDENCE(α, *s*, *n*)
8	*t* Test for Comparison of Means	see Equation 4-4, 4-5 in textbook, pg 89	see spreadsheet "8-StatisticalFunctions.xls" on course website*
9	Grubb's test	$G = \frac{\lvert x - \bar{x} \rvert}{s}$	calculate using ABS(), AVERAGE(), and STDEV()

Statistical Analysis
Linear Functions

10	Linear Regression (*m* and *b*)	see Equations 4-9, 4-10, 4-11 in textbook, pg 94	SLOPE(data_Y, data_X) INTERCEPT(data_Y, data_X)
11	Standard Deviation of Slope (s_m)	see Equations 4-12, 4-13 in textbook, pg 95	see spreadsheet "8-StatisticalFunctions.xls" on course website*
12	Confidence interval for Slope	$\mu_{m,CI} = m \pm ts_m$	calculate using TINV(α, df) and previously calculated *m* and s_m
13	Standard Error of Linear Regression		STEYX(data_Y, data_X)
14	R-squared		RSQ(data_Y, data_X)