Scientific Units and Dimensional Analysis
CH 2000: Introduction to General Chemistry I, Plymouth State University

I. INTRODUCTION

If you have previously taken any science classes, the word "units" is probably associated with the loss of points on homework or exams. However, believe it or not, units are not only necessary, but also extremely useful. First of all, units add to the vocabulary of the language of science, allowing us to communicate effectively and efficiently. Any time you report a number in science, you MUST also report a unit—fundamentally, units tell us what type of quantity is being measured or calculated. For example, if I were to ask you "How tall are you?" and you answer simply "70," I have no idea how tall you are! Are you 70 feet? 70 meters? 70 hands? or 70 inches? Or if you were to ask your parents "How long until dinner?" and they answer simply "2," do they mean 2 minutes, 2 hours, or 2 days? As you can see, reporting the units along with the number is critical to our understanding of the situation, and in fact *you already report units all the time*!

Secondly, units can be very helpful in solving problems and avoiding errors. If you learn how to use them properly, *units can help you do better on your homework, score more points on exams, and earn higher grades in your science courses*!! A systematic way of using units for your benefit is called "Dimensional Analysis."

III. THE METRIC SYSTEM

Before you go any further in this Worksheet, you may wish to re-familiarize yourself with the units used in science—specifically with the Metric System. Below is a table with some units used in the metric system and why they are used to measure. For further review, please see the Metric System Worksheet.

METRIC UNITS		
MEASUREMENT	**UNIT**	**SYMBOL**
Length	meter	m
Mass	gram	g
Volume	liter	L
Temperature	Kelvin or degree Celcius	K, °C
Number of things	mole	mol

II. ARITHMETIC WITH UNITS

The first thing to know about Dimensional Analysis is how to do arithmetic with units (or perhaps even before that, the fact that *you can do arithmetic with units*). If you know how to do simple arithmetic with algebraic variables, then you already know how to do arithmetic with units. Here is a review of simple algebraic arithmetic with "x" and "y":

1. Addition and Subtraction. (Rule: you cannot add or subtract the coefficients of different variables)
   $x + x = 2x$ $\qquad$ $x + y = x + y$
   $3x - 2x = x$ $\qquad$ $3x - 2y = 3x - 2y$
2. Multiplication and Division:

   $2x \times 3x = 6x^2$ $\qquad$ $2x \times 3y = 6xy$ $\qquad$ $4x \div 2x = 2$ $\qquad$ $4x \div 2y = 2\dfrac{x}{y}$

Examples

Perform the expressions given with the units shown and verify that you obtain the same answers:

1. 3 g + 10 g = 13 g
2. 53.0 °C – 1.2 °C = 51.8 °C
3. 2.5 m × 3 m = 7.5 m²
4. 12 L ÷ 4 L = 3
5. 15 m + 3 L = 15 m + 3 L

6. 6.1 mol × 2 = 12.2 mol
7. 65 m ÷ 5 s = 13 m/s *
8. 32 L + 12 L = 44 L
9. 60 s - 60 L = 60 s - 60 L
10. 144 J × 12s = 1728 J·s **

* Note: When one unit is divided by another, the bottom one will frequently be written using the exponent "-1." For example, "meters per second" ($\frac{m}{s}$) may be shown as m/s, or m·s⁻¹.

**Note: When 2 different units are multiplied together, they will often be separated by a dot (·).

Practice (*answers to the following questions can be found at the end of this worksheet*)

1. Find the answer to each arithmetic expression using the correct units:
   a. 12 m + 12 m =
   b. 24 s × 2 =
   c. 5 s × 5 s =
   d. 180 g – 150 g =

   e. 0.2 mole ÷ 0.2 L =
   f. 60 s – 10 s =
   g. 1.5 g × 3.5 g =
   h. 1,000 K ÷ 10 K =

III. COMPOUND UNITS

A number of quantities are measured and reported using "compound units"—that is, units made up of more than one unit. Below is a list of some common compound units used in the Metric System:

COMPOUND METRIC UNITS		
Measurement	**Unit**	**Symbol**
Area	square meters	m²
Volume	cubic meter	m³
Speed / Velocity	meters per second	$\frac{m}{s}$
Density	grams per liter, grams per cubic centimeter	$\frac{g}{L}$, $\frac{g}{cm^3}$
Concentration	moles per liter	$\frac{mol}{L}$
Molecular Weight	grams per mole	$\frac{g}{mol}$

Compound units may arise as the result of multiplying or dividing units. A common compound unit you probably use frequently is area. To find the area of a rectangle, you multiply the length by the width. For example, if your house is 20 feet wide by 50 feet long, the area (or "square footage") of your house is 20 ft × 50 ft = 1000 ft² (notice this operation is very similar to that done in Example 3 in the previous section!). All the other compound units work exactly like this, following the rules of Arithmetic with Units outlined in Section II.

Note that many compound units involve one unit divided by another. When speaking of these units, we use the word "per," as in "miles per hour." Thus, $\frac{m}{s}$ is "meters per second" and $\frac{moles}{L}$ is "moles per liter."

Examples
Perform the following operations with the units shown and verify that you obtain the same answer with the correct compound units:

1. $3 \text{ m} \times 4 \text{ m} = 12 \text{ m}^2$
2. $5 \text{ m}^2 \times 4 \text{ m} = 20 \text{ m}^3$
3. $20 \text{ m} \div 2 \text{ s} = 10 \text{ m/s}$
   (*remember: sometimes units on the bottom of the fraction will be written with a -1 exponent)

4. $0.25 \text{ mol} \div 0.5 \text{ L} = 0.5 \text{ mol/L}$
5. $3.2 \text{ g} \div 2 \text{ L} = 1.6 \text{ g/L}$
6. $125 \text{ m} \div 5 \text{ s} = 25 \text{ m·s}^{-1}$ *

Practice (*answers to the following questions can be found at the end of this worksheet*)

2. Perform the following operations with the units shown and verify that you obtain the numerical answer shown with the correct compound units:

a. $2 \text{ m} \times 3 \text{ m} \times 4 \text{ m} =$
b. $100 \text{ m}^3 \div 20 =$
c. $7.5 \text{ g} \div 2 \text{ cm}^3 =$

d. $3.5 \text{ mol} \div 0.7 \text{ L} =$
e. $0.023 \text{ g} \div 0.125 \text{ L} =$
f. $3,453 \text{ m} \div 300 \text{ s} =$

III. CANCELING UNITS (a.k.a DIMENSIONAL ANALYSIS)

One of the effects of being able to do common arithmetic on units is that *units can be canceled*. It works according to the same simple arithmetic:

$$\frac{x}{x} = 1 \qquad \text{or} \qquad x \times \frac{1}{x} = 1$$

or, with another "unit" present:

$$\frac{xy}{x} = y \qquad \text{or} \qquad xy \times \frac{1}{x} = y$$

(notice that we did something very similar in Section I, Example 4)

In fact, canceling units in this way is at the heart of Dimensional Analysis! If you are careful about writing your units and learn when and how to cancel them (as well as when *not* to cancel them), you will be able to find answers to seemingly difficult problems, check your answers, and avoid errors!

Examples
Perform the following operations, canceling the units necessitated by the arithmetic, and verify your answers, including the units.

1. $3\frac{m}{s} \times 2 \text{ s} = 6 \text{ m}$
2. $2.5 \text{ m/s} \times 60 \text{ s} = 150 \text{ m}$
3. $10\frac{mol}{L} \times 5 \text{ L} = 50 \text{ mol}$

4. $0.12\frac{g}{L} \times 0.25 \text{ L} = 0.03 \text{ g}$
5. $50 \text{ miles/hr} \times 0.5 \text{ hr} = 25 \text{ miles}$
6. $2.4 \text{ g·cm}^{-3} \times 50 \text{ cm}^3 = 120 \text{ g}$

Practice (*answers to the following questions can be found at the end of this worksheet*)

3. Perform the following operations, canceling the units necessitated by the arithmetic:

a. $30\dfrac{\text{miles}}{\text{hr}} \times 0.75\,\text{hr} \; =$

b. $125\,\text{g/mol} \times 0.75\,\text{mol} =$

c. $15\dfrac{\text{g}}{\text{cm}^3} \times 0.9\,\text{cm}^3 \; =$

d. $0.12\dfrac{\text{g}}{\text{L}} \times 0.25\,\text{L} \; =$

e. $625\,\text{m/s} \times 3600\,\text{s} =$

f. $0.025\,\text{mol·L}^{-1} \times 1.2\,\text{L} =$

IV. CONVERTING UNITS

Dimensional Analysis and Canceling Units is particularly useful when converting units. There are two different ways in which you may need to convert units:

1. Converting the prefixes of metric units (such as, converting grams to kilograms).
2. Converting between completely different units used to measure the same type of quantity (such as converting between meters and feet).

Regardless of which type of conversion you need to do, you must know the "conversion factor"—that is, if you are converting between "x" and "y", you must know how many x's equal how many y's. For example, if you are asked to change a $1 bill into quarters, you need to know that there are 4 quarters in a dollar. In other words $1 bill = 4 quarters. If you think about it like this, you will realize that you do conversions all the time! For instance, how many inches are in a foot? How many inches in 3 feet? Hours in a day? Hours in 2.5 days?

If you know the conversion factor and can write an equality (e.g., 12 inches = 1 foot), you can convert just about anything. Knowing the equality between two units, you can write them as a ratio:

$$\dfrac{12\,\text{inches}}{1\,\text{foot}} \quad \text{or} \quad \dfrac{1\,\text{foot}}{12\,\text{inches}} \qquad\qquad \dfrac{\$1}{4\,\text{quarters}} \quad \text{or} \quad \dfrac{4\,\text{quarters}}{\$1}$$

You can then multiply anything by these ratios as if you were simply multiplying by 1. In the process, if you use the correct ratio and are careful about canceling, you can convert from one unit into another. For example, if you have 36 quarters and need to know how many dollars that is:

$$36\,\text{quarters} \times \dfrac{\$1}{4\,\text{quarters}}$$

The quarters will cancel out, leaving you with only dollars ($):

$$36\,\cancel{\text{quarters}} \times \dfrac{\$1}{4\,\cancel{\text{quarters}}} = \$9$$

On the other hand, if you have $9 and wanted to know how many quarters that is, simply flip the ratio over to cancel out the other unit:

$$\$9 \times \dfrac{4\,\text{quarters}}{\$1} = 36\,\text{quarters}$$

A. CONVERTING METRIC PREFIXES

You may wish to review the prefixes of the metric system in the "Metric System Worksheet." Here are a few of the more common ones:

Metric System Prefixes			
Prefix	**Abbrev.**	**Power of ten**	**Conversion**
nano-	n	10^{-9} (one one-billionth)	1 unit = 10^9 nano-units
micro-	μ	10^{-6} (one one-millionth)	1 unit = 10^6 micro-units
milli-	m	10^{-3} (one one-thousandth)	1 unit = 1000 milli-units
centi-	c	10^{-2} (one hundredth)	1 unit = 100 centi-units
kilo-	k	10^3	1000 units = 1 kilo-unit

Notice that the forth column lists a useful equality for converting between the base unit and the pre-fix unit (Remember: nano-, micro-, milli-, and centi-units are *smaller* than the base unit, so it takes many of them to make one base unit; a kilo-unit is *larger* than a base-unit, so it takes many units to make one kilo-unit). Using these equalities to write ratios provides an easy way to convert metric units with different prefixes.

Examples
1. Verify that the following units are converted correctly by checking that the units cancel:

   a. $2\,\text{m} \times \dfrac{100\,\text{cm}}{1\,\text{m}} = 200\,\text{cm}$

   b. $4\,\text{kg} \times \dfrac{1000\,\text{g}}{1\,\text{kg}} = 4000\,\text{g}$

   c. $125\,\text{mL} \times \dfrac{1\,\text{L}}{1000\,\text{mL}} = 0.125\,\text{L}$

   d. $35\,\dfrac{\text{m}}{\text{s}} \times \dfrac{1000\,\text{mm}}{1\,\text{m}} = 35{,}000\,\dfrac{\text{mm}}{\text{s}}$

2. Convert each quantity into the units indicated:

   Answers

   a. 2.25 L into milliliters

   $2.25\,\cancel{\text{L}} \times \dfrac{1000\,\text{mL}}{1\,\cancel{\text{L}}} = 2{,}250\,\text{mL}$

   b. 34.5 g into kilograms

   $34.5\,\cancel{\text{g}} \times \dfrac{1\,\text{kg}}{1000\,\cancel{\text{g}}} = 0.0345\,\text{kg}$

   c. 0.025 mmoles into moles

   $0.025\,\cancel{\text{mmol}} \times \dfrac{1\,\text{mol}}{1000\,\cancel{\text{mmol}}} = 0.000025\,\text{mol}$

   d. 1.25 g·mol⁻¹ into centigrams per mol

   $1.25\,\dfrac{\cancel{\text{g}}}{\text{mol}} \times \dfrac{100\,\text{cg}}{1\,\cancel{\text{g}}} = 125\,\dfrac{\text{cg}}{\text{mol}}$

Practice (*answers to the following questions can be found at the end of this worksheet*)
4. Perform each of the following conversions by canceling out the units:

   a. $8\,\text{L} \times \dfrac{10^6\,\mu\text{L}}{1\,\text{L}} =$

   b. $24\,\text{cm} \times \dfrac{1\,\text{m}}{100\,\text{cm}} =$

   c. $0.675\,\text{kg} \times \dfrac{1000\,\text{g}}{1\,\text{kg}} =$

   d. $20.13\,\dfrac{\text{g}}{\text{L}} \times \dfrac{1\,\text{L}}{1000\,\text{mL}} =$

5. Convert each quantity into the units indicated:
   a. 245 cm into meters
   b. 0.034 L into mL
   c. 0.945 kg into g
   d. 845 m/s into kilometers per second

B. CONVERTING DIFFERENT TYPES OF UNITS

As mentioned above, Dimensional Analysis can help us convert between different units used to measure the same types of quantities, as long as the conversion factor is known. Below is a list of conversion factors for converting between Metric and American System units.

Various Conversions		
Type	**Metric Unit**	**American Unit**
Distance / Length	2.54 cm = 1 in 0.914 m = 3 ft = 1 yd 1.61 km = 1 mile	
Energy	1 cal = 4.184 J = 0.00396 BTU	
Mass	28.35 g = 1 oz 1 kg = 2.2 lb	
Volume	$1\ cm^3 = 1\ mL$ 29.6 mL = 1 fl oz 0.94625 L = 2 pints = 1 quart = 0.25 gal	
Time	1 hr = 60min; 1 min = 60 sec	

Examples

1. Verify that the following units are converted correctly by checking that the units cancel:

   a. $4\,hr\ \times \dfrac{60\,min}{1\,hr} = 240\,min$

   b. $10\,kg\ \times \dfrac{2.2\,lb}{1\,kg} = 22\,lb$

   c. $125\,cm\ \times \dfrac{1\,in}{2.54\,cm} = 49.21\,in$

   d. $65\dfrac{miles}{hr} \times \dfrac{1.61\,km}{1\,mile} = 104.65\dfrac{km}{hr}$

2. Convert each quantity into the units indicated:

   Answers

   a. 1.5 L into quarts

   $1.5\,\cancel{L} \times \dfrac{1\,quart}{0.94625\,\cancel{L}} = 1.58\,qt$

   b. 6.1 ft into meters

   $6.1\,\cancel{ft} \times \dfrac{0.914\,m}{3\,\cancel{ft}} = 1.86\,m$

   c. 12 fl oz into milliliters

   $12\,\cancel{fl\ oz} \times \dfrac{29.6\,mL}{1\,\cancel{fl\ oz}} = 355.2\,mL$

   d. 15 km/hr into kilometers per minute

   $15\dfrac{km}{\cancel{hr}} \times \dfrac{1\,\cancel{hr}}{60\,min} = 0.25\dfrac{km}{min}$

Practice (*answers to the following questions can be found at the end of this worksheet*)

6. Perform each of the following conversions by canceling out the units:

   a. $5\,yd\ \times \dfrac{0.914\,m}{1\,yd}\ =$

   b. $24\,J\ \times \dfrac{1\,cal}{4.184\,J}\ =$

   c. $7.5\,L\ \times \dfrac{2\,pints}{0.94625\,L}\ =$

   d. $142\dfrac{g}{mol}\ \times \dfrac{1\,oz}{28.35\,g}\ =$

7. Convert each quantity into the units indicated:
   a. 250 miles into kilometers
   b. 45 seconds into minutes
   c. 250 cal into Joules
   d. 0.55 oz/L into grams per liter

C. MULTIPLE CONVERSIONS AT ONCE

Sometimes it is necessary to do several conversions at once, including converting between different prefixes, converting compound units, or combinations of both!. These types of conversions can seem daunting at first, but by using Dimensional Analysis and being careful about writing out and canceling the units, you can perform these conversions easily and without errors.

Example 1: Question: How many centimeters is 595 nanometers?

Solution: You could use techniques you learned in the "Metric System" and "Scientific Notation" worksheet to move the decimal place, but there is a lot of room for error there. Instead, try using Dimensional Analysis. Start out by writing down your starting number and the units you want in the end (with a lot of space in between!):

$$595 \text{ nm} \qquad\qquad = \qquad\qquad \text{cm}$$

Then, write down the conversion ratios for the two prefixes to convert to the base unit:

$$\frac{100 \text{ cm}}{1 \text{ m}} \qquad\qquad \frac{1\times10^9 \text{ nm}}{1 \text{ m}}$$

Next, flip over the ratios as needed to cancel the units you are converting *from*. Write these into the original equation you started:

$$595 \text{ nm} \times \frac{1 \text{ m}}{1\times10^9 \text{ nm}} \times \frac{100 \text{ cm}}{1 \text{ m}} \qquad = \qquad\qquad \text{cm}$$

Finally, cancel the units and perform the arithmetic:

$$595 \cancel{\text{ nm}} \times \frac{1 \cancel{\text{ m}}}{1\times10^9 \cancel{\text{ nm}}} \times \frac{100 \text{ cm}}{1 \cancel{\text{ m}}} \qquad = \ 5.95\times10^{-5} \ \text{cm}$$

Example 2: Question: How many cubic meters is 50 ft^3?

Solution: Start out by writing down your starting number and the units you want in the end:

$$50 \text{ ft}^3 \qquad\qquad = \qquad\qquad \text{m}^3$$

Then, write down the conversion factors for the simpler units you know:

$$\frac{0.914 \text{ m}}{3 \text{ ft}}$$

Next, arrange the ratios as needed to cancel out the units you are converting *from*. Notice that in this case, you will need to cube the entire conversion ratio:

$$50 \text{ ft}^3 \times \left(\frac{0.914 \text{ m}}{3 \text{ ft}}\right)^3 \quad = \qquad\qquad \text{m}^3$$

Do the arithmetic on the conversion factor first:

$$\left(\frac{0.914 \text{ m}}{3 \text{ ft}}\right)^3 = \left(\frac{0.914}{3}\right)^3 \left(\frac{\text{m}}{\text{ft}}\right)^3 = (0.30467)^3 \frac{\text{m}^3}{\text{ft}^3} = 0.02828 \frac{\text{m}^3}{\text{ft}^3}$$

Finally, perform the arithmetic, making sure the units cancel:

$$50 \cancel{\text{ft}^3} \times 0.02828 \frac{\text{m}^3}{\cancel{\text{ft}^3}} = 1.414 \text{ m}^3$$

Example 3: Question: How many miles per hour is 500 m/s?

Solution: Start out by writing down your starting number and the units you want in the end:

$$500 \text{ m/s} \qquad = \qquad \text{mi/hr}$$

Then, write down the conversion factors you will need. Concentrate on one type of unit at a time:

Distance: $\dfrac{1 \text{ mi}}{1.61 \text{ km}} \qquad \dfrac{1000 \text{ m}}{1 \text{ km}}$

time: $\dfrac{60 \text{ s}}{1 \text{ min}} \qquad \dfrac{1 \text{ hr}}{60 \text{ min}}$

Next, arrange the ratios as needed to cancel out the units you are converting *from*. Notice that you may have to flip some ratios over:

$$500 \frac{\text{m}}{\text{s}} \times \frac{1 \text{ km}}{1000 \text{ m}} \times \frac{1 \text{ mi}}{1.61 \text{ km}} \times \frac{60 \text{ s}}{1 \text{ min}} \times \frac{60 \text{ min}}{1 \text{ hr}} \qquad = \qquad \text{mi/hr}$$

Finally, perform the arithmetic, making sure the units cancel:

$$500 \frac{\cancel{\text{m}}}{\cancel{\text{s}}} \times \frac{1 \cancel{\text{km}}}{1000 \cancel{\text{m}}} \times \frac{1 \text{ mi}}{1.61 \cancel{\text{km}}} \times \frac{60 \cancel{\text{s}}}{1 \cancel{\text{min}}} \times \frac{60 \cancel{\text{min}}}{1 \text{ hr}} \qquad = \qquad 1118 \text{ mi/hr}$$

Practice (*answers to the following questions can be found at the end of this worksheet*)

8. Convert each quantity into the units indicated:
   a. 5.3 kg/mole into grams per millimole
   b. 96 square inches into square centimeters
   c. 2.5 g/L into ounces per quart
   d. 8.5 g/L into grams per cubic inch
   e. 65 mi/hr into meters per min

ANSWERS TO **PRACTICE** QUESTIONS

1. a. 24 m   b. 48 s   c. 25 s^2 d. 30 g   e. 1 mol/L   f. 50 m/s   g. 5.25 g^2 h. 100
2. a. 24 m^3 b. 5 m^2 c. 3.75 g/cm^3 d. 5 mol/L   e. 0.184 g/L   f. 11.51 m/s
3. a. 22.5 miles b. 93.75 g   c. 13.5 g   d. 0.03 g   e. 2,250,000 m   f. 0.03 mol
4. a. 8×10^6 µL b. 0.024 m   c. 675 g   d. 0.02013 g/mL
5. a. 2.45 m   b. 34 mL   c. 945 g   d. 0.845 km/s
6. a. 4.57 m   b. 5.74 cal   c. 15.85 pints d. 5.01 oz/mol
7. a. 402.5 km   b. 0.75 s   c. 1046 J   d. 15.59 g/L

8. a. $\quad 5.3\,\dfrac{\cancel{kg}}{mol}\times\dfrac{1000\,g}{1\,\cancel{kg}}\times\dfrac{1\,\cancel{mol}}{1000\,mmol}=5.3\,\dfrac{g}{mmol}$

  b. $\quad 96\,in^2\times\left(\dfrac{2.54\,cm}{1\,in}\right)^2=96\,\cancel{in^2}\times\left(\dfrac{6.4516\,cm^2}{1\,\cancel{in^2}}\right)=619.3\,cm^2$

  c. $\quad 2.5\,\dfrac{g}{\cancel{L}}\times\dfrac{1\,oz}{28.35\,\cancel{g}}\times\dfrac{0.94625\,\cancel{L}}{1\,qt}=0.2086\,\dfrac{oz}{qt}$

  d.

$8.5\,\dfrac{g}{L}\times\dfrac{1\,L}{1000\,mL}\times\dfrac{1\,mL}{1\,cm^3}\times\left(\dfrac{2.54\,cm}{1\,in}\right)^3=8.5\,\dfrac{g}{\cancel{L}}\times\dfrac{1\,\cancel{L}}{1000\,\cancel{mL}}\times\dfrac{1\,\cancel{mL}}{1\,\cancel{cm^3}}\times\dfrac{16.387064\,\cancel{cm^3}}{1\,in^3}=0.1393\,\dfrac{g}{in^3}$

  e. $\quad 65\,\dfrac{mi}{hr}\times\dfrac{1.61\,km}{1\,mi}\times\dfrac{1000\,m}{1\,km}\times\dfrac{1\,hr}{60\,min}=65\,\dfrac{\cancel{mi}}{\cancel{hr}}\times\dfrac{1.61\,\cancel{km}}{1\,\cancel{mi}}\times\dfrac{1000\,m}{1\,\cancel{km}}\times\dfrac{1\,\cancel{hr}}{60\,min}=1744\,\dfrac{m}{min}$

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